Sampling Subgraphs with Guaranteed Treewidth for Accurate and Efficient Graphical Inference

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OVERVIEW

- **Given** a large real-world graph $G = (V, E)$
- **Problem**: to sample a subgraph $U = (V, E')$ such that $E' \subseteq E$, which preserves the properties of $G$
- **Main idea**: to bound the treewidth of $U$ to $k$
- **Source codes**: https://datalab.snu.ac.kr/btw

MOTIVATION

Graphical Inference
- Essential task for solving node classification
- Loopy belief propagation (LBP)
- Fast but runs approximate inference
- The junction tree algorithm (JT)
- Slow but runs exact inference
- Exponential complexity with treewidth ($\mathcal{G}$)
- Treewidth: how much $\mathcal{G}$ resembles a tree

Research Motivation
- Can we sample subgraphs with bounded TW?
  - JT on the subgraphs → **accurate** classification
  - LBP on the subgraphs → **fast** classification

PROPOSED APPROACH

Bounded Treewidth Sampling (BTW)
- Bounds the treewidth of subgraphs with $k$
- Maintains a $k$-tree $K$ during a sampling process
- Use a score function $m$ for sampling edges

Step 1: Initialization
- Given a graph $G$, BTW generate two graphs
  - A subgraph $U_{k+1}$ having $k + 1$ nodes
  - A $k$-tree $K_{k+1}$ which is a complete graph

Step 2: Incremental Updates
- Define a score function $m(u, C_k)$
  - $u$ is a new node, and $C_k$ is a $k$-clique in $K_{k+1}$
- Select $(u_{k+2}, C_k^*) = \arg\max_{u, C_k} m(u, C_k)$
- Connect $u_{k+2}$ to $C_k^*$ in both $U_{k+1}$ and $K_{k+1}$

Experimental Setup
- Sample subgraphs using BTW (or others)
- Divide the labels for the $k$-fold validation
- Run JT or LBP for node classification
- **Datasets**: 4 real graphs with 2 – 16 labels
- **Evaluation**: classification accuracy & time

Effects for Node Classification

Comparison with Other Algorithms (by LBP)

<table>
<thead>
<tr>
<th>Method</th>
<th>Wikipedia</th>
<th>CoRA</th>
<th>PubMed</th>
<th>PolBlogs</th>
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</thead>
<tbody>
<tr>
<td>RE</td>
<td>35.3 ± 0.2</td>
<td>57.9 ± 0.3</td>
<td>61.8 ± 0.4</td>
<td>75.8 ± 1.0</td>
</tr>
<tr>
<td>RNE</td>
<td>51.3 ± 0.3</td>
<td>65.2 ± 0.2</td>
<td>71.1 ± 0.1</td>
<td>84.4 ± 0.6</td>
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<tr>
<td>HYB</td>
<td>49.4 ± 0.2</td>
<td>64.5 ± 0.2</td>
<td>69.8 ± 0.3</td>
<td>83.4 ± 0.4</td>
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<tr>
<td>RW</td>
<td>26.5 ± 2.7</td>
<td>43.0 ± 1.7</td>
<td>56.5 ± 1.2</td>
<td>65.2 ± 3.4</td>
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<tr>
<td>RJ</td>
<td>36.6 ± 0.4</td>
<td>55.4 ± 0.3</td>
<td>63.2 ± 0.4</td>
<td>75.6 ± 0.5</td>
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<td>FS</td>
<td>29.8 ± 0.2</td>
<td>47.9 ± 0.2</td>
<td>56.2 ± 0.5</td>
<td>72.4 ± 0.8</td>
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<tr>
<td>FF</td>
<td>49.4 ± 0.2</td>
<td>63.7 ± 0.3</td>
<td>62.8 ± 0.4</td>
<td>79.8 ± 0.9</td>
</tr>
<tr>
<td>BTW</td>
<td>56.1 ± 0.5</td>
<td>68.6 ± 0.3</td>
<td>74.8 ± 0.4</td>
<td>86.6 ± 0.9</td>
</tr>
</tbody>
</table>

BEST: 66.0 ± 0.0 | 86.6 ± 0.0 | 86.6 ± 0.0 | 86.6 ± 0.0

Best trade-off of acc. and speed: BEST

Classifiction accuracy